9-3 day 2 Taylor's Theorem

Learning Objectives:

I can determine the number of terms needed in a Taylor (or MacLaurin) polynomial that are needed to get the desired accuracy in making an approximation.

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Ex1. Let $P_n(x)$ be the n^{th} degree MacLaurin polynomial for f(x)=cos(x). Find the degree polynomial needed to have an error less than $1x10^{-5}$ when we approximate f(.2).

error

$x=.2$

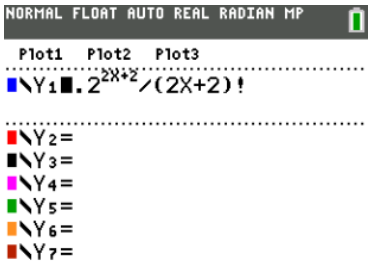
$n=0$ $n=1$ $n=2$

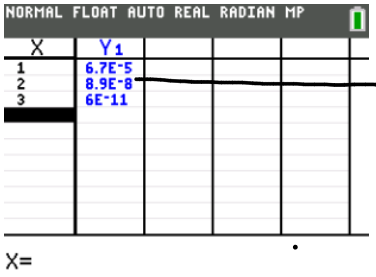
$$\cos x \Rightarrow 1-\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots+\frac{(-1)^n x^{2n}}{(2n)!}$$

$$\text{error} < \left|\frac{(-1)^{n+1}\cdot x^{2(n+1)}}{(2n+2)!}\right|$$

$$\text{error} < \frac{x^{2n+2}}{(2n+2)!} \qquad \frac{x^{2n+2}}{(2n+2)!} < 1x10^{-5}$$

$$\frac{(.2)^{2n+2}}{(2n+2)!} < 1x10^{-5}$$

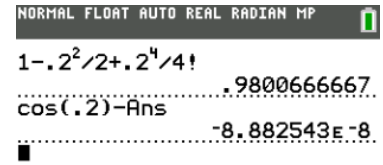




$n=2$

4th degree

$$P_4 = 1-\frac{x^2}{2!}+\frac{x^4}{4!}$$



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Ex2. Determine the degree of the Taylor polynomial $P_n(x)$ expanded about x=0 that should be used to approximate $y=e^x$ with an error of less than .0001 on [-.5,.5].

1×10^{-4}

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

$$\text{error} = \frac{f^{n+1}(c)\,|b-a|^{n+1}}{(n+1)!}$$

error $\dfrac{2\,|.5-0|^{n+1}}{(n+1)!}$

e^x

0 $\quad$ $\frac{1}{2}$

$e^{1/2} = \sqrt{e}$

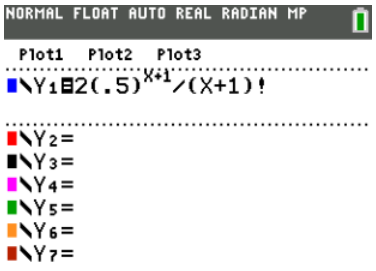
$\dfrac{2(.5)^{n+1}}{(n+1)!} < 1 \times 10^{-4}$

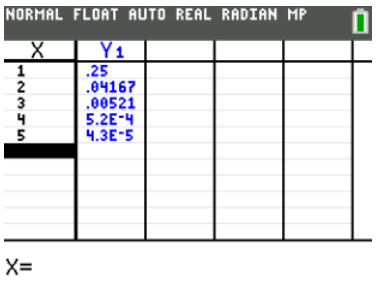
$2 < e < 3$

$\sqrt{2} < \sqrt{e} < \sqrt{3} < \sqrt{4}$

$\sqrt{e} < 2$

$n = 5$





5th degree

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Homework

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