

9-3 day 2 Taylor's Theorem

Learning Objectives:

I can determine the number of terms needed in a Taylor (or MacLaurin) polynomial that are needed to get the desired accuracy in making an approximation.

Ex1. Let $P_n(x)$ be the n^{th} degree Maclaurin polynomial for $f(x)=\cos(x)$. Find the degree polynomial needed to have an error less than 1×10^{-5} when we approximate

$f(.2)$ $x=.2$

$$\cos x \Rightarrow 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\text{error} < \left| \frac{(-1)^{n+1} x^{2(n+1)}}{(2n+2)!} \right|$$

$$\text{error} < \frac{x^{2n+2}}{(2n+2)!} \quad \frac{x^{2n+2}}{(2n+2)!} < 1 \times 10^{-5}$$

$$\frac{(.2)^{2n+2}}{(2n+2)!} < 1 \times 10^{-5}$$

NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

Y1 = .2^{2X+2} / (2X+2)!

Y2 =

Y3 =

Y4 =

Y5 =

Y6 =

Y7 =

NORMAL FLOAT AUTO REAL RADIAN MP

TABLE SETUP

TblStart = 4

ΔTbl = 1

Indent: Auto Ask

Depend: Auto Ask

X	Y1			
1	6.7E-5			
2	8.9E-8			
3	6E-11			

→ n=2

4th degree

$$P_4 = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

NORMAL FLOAT AUTO REAL RADIAN MP

1 - .2² / 2 + .2⁴ / 4!

.9800666667

cos(.2) - Ans

-8.882543E-8

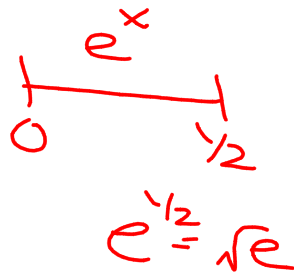
Ex2. Determine the degree of the Taylor polynomial

$P_n(x)$ expanded about $x=0$ that should be used to approximate $y=e^x$ with an error of less than $.0001$ on $[-.5,.5]$. 1×10^{-4}

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

$$\text{error} = \frac{f^{(n+1)}(c) |b-a|^{n+1}}{(n+1)!}$$

$$\text{error} = \frac{2 |.5-0|^{n+1}}{(n+1)!}$$



$$\frac{2(.5)^{n+1}}{(n+1)!} < 1 \times 10^{-4}$$

$$2 < e < 3$$

$$\sqrt{2} < \sqrt{e} < \sqrt{3} < \sqrt{4}$$

$$\sqrt{e} < 2$$

$$n=5$$

NORMAL FLOAT AUTO REAL RADI AN MP

Plot1 Plot2 Plot3

Y1 = 2(.5)^(X+1)/(X+1)!

Y2 =

Y3 =

Y4 =

Y5 =

Y6 =

Y7 =

NORMAL FLOAT AUTO REAL RADI AN MP

X	Y1				
1	.25				
2	.04167				
3	.00521				
4	5.2E-4				
5	4.3E-5				

X=

5th degree

Homework

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